MATH 101 V2A – Homework

Solutions

February 2nd

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1. Explain why the calculation

$$\int_{-1}^{1} \frac{1}{t^2}\, dt = \frac{-1}{t}\Big|_{-1}^{1} = -2$$

is invalid.

Solution: The above calculation uses the FTC to say that the integral can be evaluated using an antiderivative of $\frac{1}{t^2}$, but in order to use the FTC, $\frac{1}{t^2}$ needs to continuous on $[-1, 1]$, which it's not.

2. Find an example of a function $f(t)$ such that the integral $\displaystyle\int_{-1}^{1} f(t)dt$ is a convergent, improper integral.

Solution: The function $f(t) = \frac{1}{\sqrt{|t|}}$ will work because

$$\begin{aligned}
\lim_{T_1 \to 0^-} \int_{-1}^{T_1} \frac{1}{\sqrt{|t|}} dt &= \lim_{T_1 \to 0^-} \int_{-1}^{T_1} \frac{1}{\sqrt{-t}} dt \\
&= \lim_{T_1 \to 0^-} \left(-2\sqrt{-t} \Big|_{-1}^{T_1} \right) \\
&= \lim_{T_1 \to 0^-} \left(-2\sqrt{-T_1} + 2 \right) \\
&= 2.
\end{aligned}$$

Similarly,

$$\lim_{T_2 \to 0^+} \int_{T_2}^{1} \frac{1}{\sqrt{|t|}} dt = \lim_{T_2 \to 0^+} \int_{T_2}^{1} \frac{1}{\sqrt{t}} dt = 2.$$

Therefore

$$\int_{-1}^{1} \frac{1}{\sqrt{|t|}} dt = 4.$$