MATH 101 V2A – Homework

Solutions

January 19th

Let

$$f(x) = \frac{1}{(2x+1)^2},$$
$$g(x) = x\sqrt{1+7x^2} + 1,$$
$$h(x) = xe^{x^2} + 2.$$

Calculate the area from $x = 0$ to $x = 1$ between the graphs of:

1. $h(x)$ and $g(x)$.
2. $h(x)$ and $f(x)$.

Solution: (i) In the video we showed that $\int_0^1 h(x)dx = \frac{1}{2}(e-1) + 2$. The area between the graphs of $h(x)$ and $g(x)$ will be

$$\int_0^1 h(x)dx - \int_0^1 g(x)dx.$$

So, we need to calculate the second integral.

$$\int_0^1 x\sqrt{1+7x^2} + 1dx = 1 + \int_0^1 x\sqrt{1+7x^2}dx.$$

Let $u = 1 + 7x^2$. Then $du = 14xdx$, so

$$\begin{aligned}\int_0^1 x\sqrt{1+7x^2}dx &= \int_1^8 \frac{1}{14}\sqrt{u}du \\ &= \frac{1}{14} \cdot \frac{2}{3} u^{3/2}\Big|_1^8 \\ &= \frac{16\sqrt{2}}{21}.\end{aligned}$$

Therefore the area between the graphs of $h(x)$ and $g(x)$ is $\frac{1}{2}(e-1) + 2 - \frac{16\sqrt{2}}{21} - 1$.

(ii) Similarly, we need to calculate $\int_0^1 f(x)dx$. Let $u = 2x+1$. Then $du = 2dx$ and

$$\begin{aligned}\int_0^1 \frac{1}{(2x+1)^2}dx &= \frac{1}{2}\int_1^3 \frac{1}{u^2}du \\ &= \frac{1}{2}\left(-\frac{1}{u}\Big|_1^3\right) \\ &= \frac{1}{2}(-\frac{1}{3}+1) \\ &= \frac{1}{3}.\end{aligned}$$

Therefore the are between the graphs of $h(x)$ and $f(x)$ is $\frac{1}{2}(e-1)+2-\frac{1}{3}$.