This examination has 8 pages of questions excluding this cover

The University of British Columbia
Midterm 2 - March 12, 2014

Mathematics 103: Integral Calculus with Applications to Life Sciences

202 (Carlquist), 203 (Kim), 206 (Bruni), 207 (Namazi), 208 (Wong), 209 (Wardil)

Closed book examination

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<tr>
<th>Last Name: ________________</th>
<th>First Name: ________________</th>
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<td>Student Number: ____________</td>
<td>Section: circle above</td>
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Rules governing examinations:

1. No books, notes, electronic devices or any papers are allowed. To do your scratch work, use the back of the examination booklet. Additional paper is available upon request.
2. You should be prepared to produce your library/AMS card upon request.
3. No student shall be permitted to enter the examination room after 10 minutes or to leave before the completion of the examination.
4. You are not allowed to communicate with other students during the examination. Students may not purposely view other’s written work nor purposefully expose his/her own work to the view of others or any imaging device.
5. At the end of the exam, you will put away all writing implements upon instruction. Students will continue to follow all of the above rules while the papers are being collected.
6. Students must follow all instructions provided by the invigilator.
7. Students are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions
8. Any deviation from these rules will be treated as an academic misconduct. The plea of accident or forgetfulness shall not be received.

I agree to follow the rules outlined above __________________________

(signature)

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<th>Question:</th>
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Important

1. Simplify all your answers as much as possible and express answers in terms of fractions or constants such as $\sqrt{e}$ or $\ln(4)$ rather than decimals.
2. Show all your work and explain your reasonings clearly!
3. Questions in a section are weighted evenly unless otherwise stated.
1. (15 points) Evaluate the following integrals
   (Full marks for correct answer. Work must be shown for partial marks. Simplify fully.)

   a. (3 points) \( I_1 = \int \frac{4x + x^2}{x} \, dx \)

   ANSWER: \( I_1 = \) ________________

   b. (3 points) \( I_2 = \int \frac{e^x}{e^x + 1} \, dx \)

   ANSWER: \( I_2 = \) ________________
c. (3 points) \( I_3 = \int x^{10} \ln(x) \, dx \)

ANSWER: \( I_3 = \) 

\[ I_4 = \int_{\pi/2}^{0} \frac{\sin x}{2 - \sin^2 x} \, dx \)

ANSWER: \( I_4 = \) 

e. (3 points) \( I_5 = \int_1^4 \frac{4}{4x + x^2} dx \)

ANSWER: \( I_5 = \) ________________________________
2. (6 points) (Work must be shown for partial or full marks. Simplify fully.)

In a certain UBC midterm exam, students are given 1 hour to complete the paper. Each student is allowed to hand in his/her paper and leave the exam hall at any point of time. Let \( t \) represent the amount of time (in hours) that has passed since the start of the exam. The probability distribution of the various timings at which students choose to leave the exam hall is represented by the following probability density function:

\[
p(t) = k(t - t^3), \quad \text{for } 0 \leq t \leq 1
\]

where \( k \) is a positive number.

a. (2 points) Determine the value of \( k \) that makes \( p(t) \) a probability density function.

b. (2 points) What is the probability that a student leaves the exam hall within the first 30 minutes?

c. (2 points) What is the average (mean) amount of time (in hours) that a student spends on the exam?
3. (6 points) (Work must be shown for partial or full marks. Simplify fully.)

Wile E. Coyote is trying to catch the Road Runner using his recently purchased ACME rocket-propelled car. His velocity is given by $v(t) = t^2 - 2t$ kilometres per hour where $t$ represents the time in hours.

a. (3 points) Find the total displacement between $t = 1$ to $t = 3$.

b. (3 points) Gas is quite expensive for his rocket-powered car. If Wile E. Coyote pays 10 dollars for every kilometre traveled, how much is he paying over the 2 hour period between $t = 1$ and $t = 3$?
4. (6 points) (Work must be shown for partial or full marks. Simplify fully.)

A thin piece of metal of length 9 centimetres is positioned on the line between \( x = 1 \) and \( x = 10 \). The mass density per unit of length is \( \rho(x) = \frac{\ln x}{x} \) grams per centimetre, \( 1 \leq x \leq 10 \).

a. (3 points) Find the total mass of the piece.

b. (3 points) Compute the centre of mass of the piece.
5. (7 points) (Work must be shown for partial or full marks. Simplify fully.)

Consider the region of the plane given by $0 < x \leq 1$ and $0 \leq y \leq \frac{1}{\sqrt{x}}$.

a. (1 point) Sketch the region.

b. (3 points) Write the improper integral that represents the area of this region. Compute the area of the region if the integral is convergent and show it is divergent otherwise.

c. (3 points) Consider the 3-dimensional solid obtained by rotating the above region about the $y$-axis. Write the improper integral that represents the volume of this solid. Compute the volume if the integral is convergent and show it is divergent otherwise.
Formulas

You can use this page and its back for scratch work.

**Trigonometric identities**

\[
\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta; \quad \text{for } \alpha = \beta: \quad \sin(2\alpha) = 2 \sin \alpha \cos \alpha
\]

\[
\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta; \quad \text{for } \alpha = \beta: \quad \cos(2\alpha) = 2 \cos^2 \alpha - 1
\]

\[
\sin^2 \alpha + \cos^2 \alpha = 1, \quad \tan^2 \alpha + 1 = \sec^2 \alpha = \frac{1}{\cos^2 \alpha}
\]

**Some useful trigonometric values**

\[
\sin(0) = 0, \quad \sin \left(\frac{\pi}{6}\right) = \frac{1}{2}, \quad \sin \left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}, \quad \sin \left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}, \quad \sin \left(\frac{\pi}{2}\right) = 1, \quad \sin(\pi) = 0
\]

\[
\cos(0) = 1, \quad \cos \left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}, \quad \cos \left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}, \quad \cos \left(\frac{\pi}{3}\right) = \frac{1}{2}, \quad \cos \left(\frac{\pi}{2}\right) = 0, \quad \cos(\pi) = -1
\]

\[
\tan(0) = 0, \quad \tan \left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}}, \quad \tan \left(\frac{\pi}{4}\right) = 1, \quad \tan \left(\frac{\pi}{3}\right) = \sqrt{3}, \quad \tan(\pi) = 0
\]

**Derivatives**

\[
\frac{d}{dx} \arcsin x = \frac{1}{\sqrt{1-x^2}}, \quad \frac{d}{dx} \arccos x = -\frac{1}{\sqrt{1-x^2}}, \quad \frac{d}{dx} \arctan x = \frac{1}{1+x^2}
\]